SYDNEY GRAMMAR SCHOOL

NAME ______________________

MATHS MASTER ____________

CANDIDATE NUMBER

2023 Trial Examination

Form VI Mathematics Advanced

Tuesday 8th August 2023
8:40am

General Instructions

- Reading time — 10 minutes
- Working time — 3 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.
- Remove the central staple: you should have this cover booklet with Section I, 4 booklets for Section II, and the multiple-choice answer sheet.

Total Marks: 100

Section I (10 marks) Questions 1 – 10

- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.

Section II (90 marks) Questions 11 – 37

- Relevant mathematical reasoning and calculations are required.
- Answer the questions in this paper in the spaces provided.
- This section is divided into four parts. Extra writing paper is provided at the end of each part.

Collection

- Your name and master should only be written on this page.
- Write your candidate number on this page, on the start of the separate section and on the multiple choice sheet.
- Place everything inside this question booklet.

Checklist

- Reference sheet
- Multiple-choice answer sheet
- Candidature: 91 pupils

Writer: OMD

	Marks
Multiple Choice	
Part A	
Part B	
Part C	
Part D	
TOTAL	

Section I

Questions in this section are multiple-choice.
Record the single best answer for each question on the provided answer sheet.

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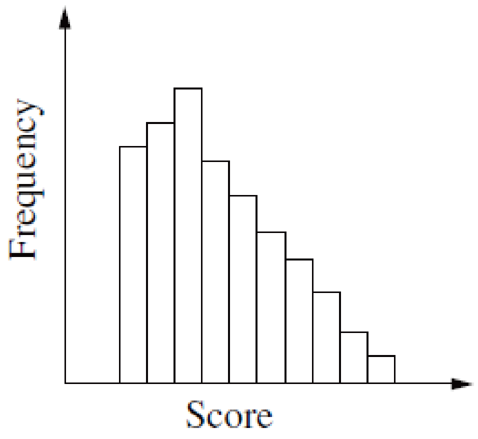
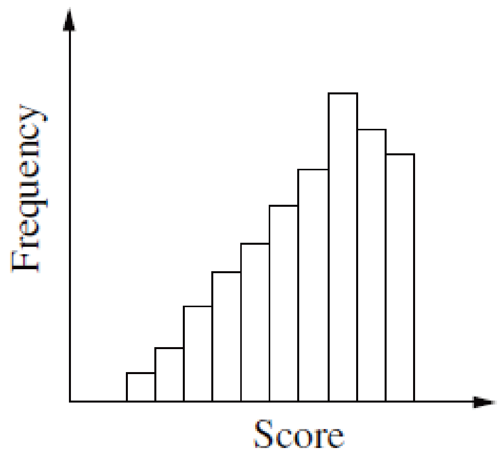
1. Which histogram best represents a dataset that is positively skewed?

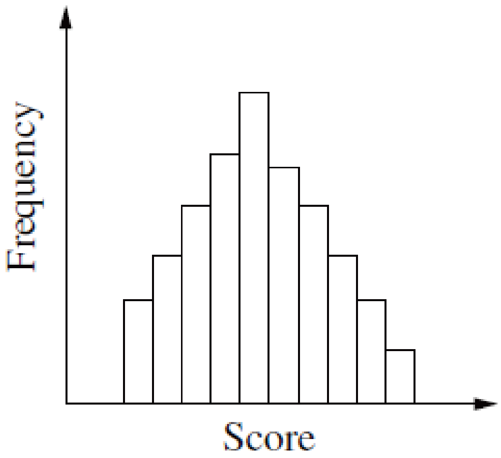
(A)



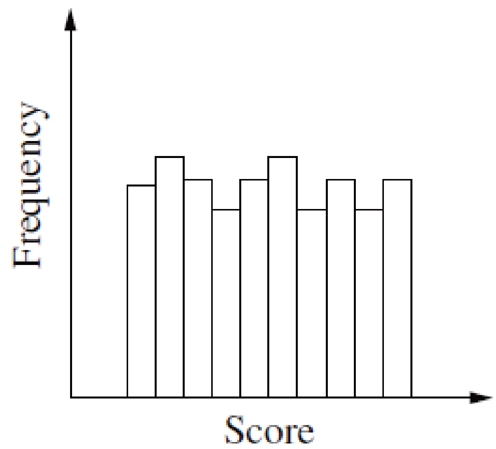
(B)



(C)



(D)



2. What is the gradient of the line passing through $A(3,0)$ and $B(0,4)$?

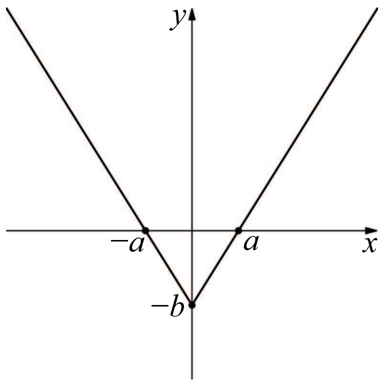
(A) $-\frac{4}{3}$

(B) $-\frac{3}{4}$

(C) $\frac{3}{4}$

(D) $\frac{4}{3}$

3. Which type of symmetry best describes the function sketched below?



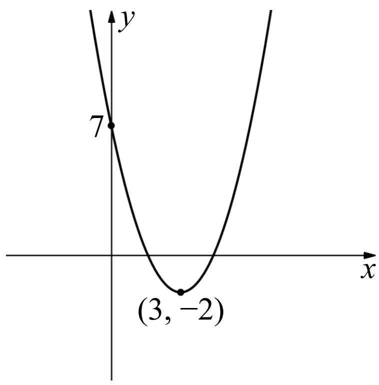
(A) Odd

(B) Even

(C) Neither

(D) Further information required

4. Which equation best describes the quadratic function sketched below?



(A) $y = (x + 3)^2 + 2$

(B) $y = (x + 3)^2 - 2$

(C) $y = (x - 3)^2 + 2$

(D) $y = (x - 3)^2 - 2$

5. In twelve years, the future value of an investment will be \$250 000. The interest rate is 6% per annum, compounded half-yearly.

   Which equation will give the present value (PV) of the investment?

   (A) $PV = \dfrac{250\,000}{(1+0.06)^{24}}$

   (B) $PV = \dfrac{250\,000}{(1+0.03)^{24}}$

   (C) $PV = \dfrac{250\,000}{(1+0.06)^{12}}$

   (D) $PV = \dfrac{250\,000}{(1+0.03)^{12}}$

6. Which transformations best describe the change from $y = f(x)$ to $y = -f(x+5)$?

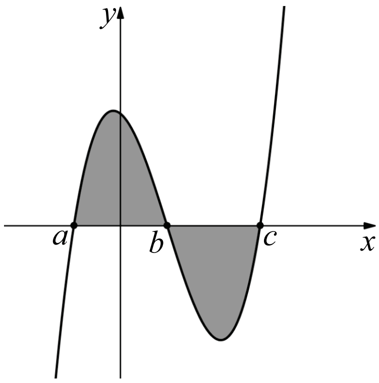
   (A) A translation left 5 units then a reflection in the y-axis.

   (B) A translation right 5 units then a reflection in the y-axis.

   (C) A translation left 5 units then a reflection in the x-axis.

   (D) A translation right 5 units then a reflection in the x-axis.

7. For the given function $y = f(x)$ sketched below, which integral does **NOT** give the shaded area?



(A) $\displaystyle\int_a^b f(x)\,dx + \int_c^b f(x)\,dx$

(B) $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

(C) $\displaystyle\int_a^b f(x)\,dx - \int_b^c f(x)\,dx$

(D) $\displaystyle\int_a^b f(x)\,dx + \left|\int_b^c f(x)\,dx\right|$

8. For the discrete random variable X, it is known that $\mathrm{E}(X) = 3$ and $\mathrm{Var}(X) = 2$. Which option below gives the correct expected value and variance of $X + 3$?

(A) $\mathrm{E}(X+3) = 3$ and $\mathrm{Var}(X+3) = 2$.

(B) $\mathrm{E}(X+3) = 6$ and $\mathrm{Var}(X+3) = 2$.

(C) $\mathrm{E}(X+3) = 3$ and $\mathrm{Var}(X+3) = 5$.

(D) $\mathrm{E}(X+3) = 6$ and $\mathrm{Var}(X+3) = 5$.

9. Which is a correct trigonometric identity?

   (A) $(1 + \sec\theta)(1 - \sec\theta) = \cot^2\theta$

   (B) $(1 + \sec\theta)(1 - \sec\theta) = \tan^2\theta$

   (C) $(1 + \sec\theta)(1 - \sec\theta) = -\cot^2\theta$

   (D) $(1 + \sec\theta)(1 - \sec\theta) = -\tan^2\theta$

10. A region in the plane is bounded by the curve $y = \dfrac{1}{3x}$, the x-axis, the line $x = m$, and the line $x = 2m$, where $m > 0$. Which description most accurately describes the area of this region?

    (A) The area increases as m increases.

    (B) The area decreases as m increases.

    (C) The area increases as m decreases.

    (D) The area is independent of m.

End of Section I

The paper continues in the next section

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- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

Question Nine

A ◯ B ◯ C ◯ D ◯

Question Ten

A ◯ B ◯ C ◯ D ◯

CANDIDATE NUMBER

Section II

Part A

— Remember to fill in your candidate number above —

QUESTION ELEVEN (2 marks) **Marks**

Find the equation of the line perpendicular to $y = 2x - 1$ passing through the point $A(0, -7)$. 2

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QUESTION TWELVE (2 marks) **Marks**

Classify each relation as one-to-one, many-to-one, one-to-many, or many-to-many.

(a) $y = 3x - 4$ 1

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(b) $x^2 + y^2 = 25$ 1

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QUESTION THIRTEEN (1 mark) **Marks**

Differentiate $f(x) = x^2(2x - 1)$. 1

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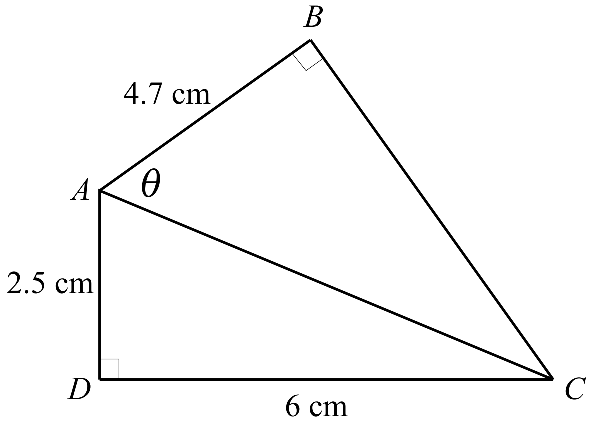
QUESTION FOURTEEN (3 marks) **Marks**

Two right-angled triangles, ABC and ADC, are shown below. 3



Calculate the size of angle θ. Give your answer correct to the nearest minute.

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QUESTION FIFTEEN (2 marks) **Marks**

Suppose that $f(x) = 2x^2$ and $g(x) = x - 1$.

(a) Find the value of $g(f\,(4))$. 1

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(b) Find $f(g\,(x))$. 1

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QUESTION SIXTEEN (2 marks) **Marks**

Find a formula for the nth term of the following sequences:

(a) $10, 12, 14, 16, \ldots$ 1

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(b) $\frac{1}{3}, 1, 3, 9, \ldots$ 1

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QUESTION SEVENTEEN (2 marks) **Marks**

Differentiate $y = (5x-3)^2$. 2

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QUESTION EIGHTEEN (2 marks) **Marks**

The gradient function of a curve is given by $f'(x) = 2x$. Given that the curve $y = f(x)$ goes through the point $A(2, -1)$, find the equation of the curve, $y = f(x)$. 2

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QUESTION NINETEEN (4 marks) **Marks**

Daniel inherits \$50 000 and invests it in an account earning interest at a rate of 0.6% per month. Each month, immediately after the interest has been paid, Daniel withdraws \$700.

The amount in the account immediately after the nth withdrawal can be determined using the recurrence relation

$$A_n = A_{n-1}(1.006) - 700\,,$$

where $n = 1, 2, 3, \ldots$ and $A_0 = 50\,000$.

(a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal. 2

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(b) Calculate the amount of interest earned in the first three months. 2

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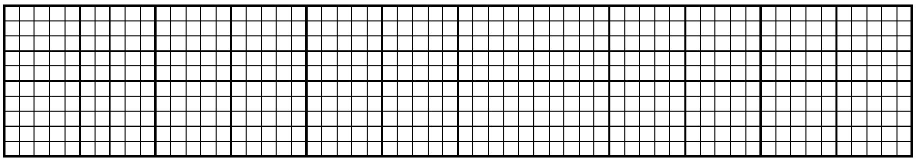
QUESTION TWENTY (4 marks) **Marks**

The Jawas run a trading outpost on Tatooine. Their accountant records their monthly sales, in millions of galactic credits, listed below:

$$3, 4, 7, 9, 11, 12, 13, 13, 14, 20, 25\,.$$

(a) On the graph paper below, draw a box plot to represent this data. [2]



(b) Identify any outliers in this data. [1]

(c) The Jawas claim that for 75% of the months, the amount received in sales is greater than 10 million galactic credits. Comment on this claim, justifying your answer. [1]

CANDIDATE NUMBER

Section II

Part B

— *Remember to fill in your candidate number above* —

QUESTION TWENTY-ONE (3 marks) **Marks**

(a) Find the equation of the tangent to the curve $y = e^{x+2}$ at the point $A(-2, 1)$. **2**

(b) Hence find point B, where the tangent in Part (a) crosses the x-axis. **1**

QUESTION TWENTY-TWO (3 marks) **Marks**

Solve the equation $9^x - 3^{x+1} = -2$, leave your answers in exact form. 3

QUESTION TWENTY-THREE (2 marks) **Marks**

Find the sum of the first 100 positive odd numbers. 2

QUESTION TWENTY-FOUR (2 marks) **Marks**

Differentiate $y = \dfrac{x^2+1}{e^{2x}}$, leave your answer in simplest form. 2

QUESTION TWENTY-FIVE (4 marks) **Marks**

The table shows the future value of an annuity of \$1.

Future values of an annuity of \$1

Years	Interest Rate per Annum			
	1%	2%	3%	4%
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

Harper is saving for a trip and estimates she will need \$20 000. She opens an account earning 4% per annum, compounded annually.

(a) How much does Harper need to deposit every year if she wishes to have enough money for the trip in 5 years time? 2

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(b) How much interest will Harper earn on her investment over the 5 years? Give your answer correct to the nearest dollar. 2

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QUESTION TWENTY-SIX (3 marks) **Marks**

(a) Find $\displaystyle\int e^{3-2x}\,dx$. 1

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(b) Evaluate $\displaystyle\int_0^1 \frac{1}{3x+1}\,dx$, leave your answer in exact form. 2

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QUESTION TWENTY-SEVEN (3 marks) **Marks**

Gizka have proven to be a pest throughout the galaxy due to their exponential rate of reproduction. The number, G, of gizka at a time, t, months after they were introduced to the planet of Tatooine can be calculated by the formula $G = 50e^{1.2t}$.

(a) How many gizka were first released on Tatooine? 1

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(b) Show that $\dfrac{dG}{dt} = 1.2G$ and hence find the rate at which the number of gizka was increasing when there were 200 gizka. 2

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QUESTION TWENTY-EIGHT (2 marks) **Marks**

(a) Differentiate $\cos^3 x$ with respect to x. 1

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(b) Hence, or otherwise, find $\displaystyle\int \sin x \cos^2 x \, dx$. 1

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CANDIDATE NUMBER

Section II

Part C

— *Remember to fill in your candidate number above* —

QUESTION TWENTY-NINE (5 marks) **Marks**

A bowl of fruit contains 18 apples of which 11 are red and 7 are green.

Matthew takes one apple at random and eats it. Evelyn then takes an apple at random and eats it.

(a) By drawing a probability tree diagram, or otherwise, find the probability that Matthew and Evelyn eat apples of the same colour. 3

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(b) Given that Matthew and Evelyn eat apples of the same colour, find the probability that they are red. 2

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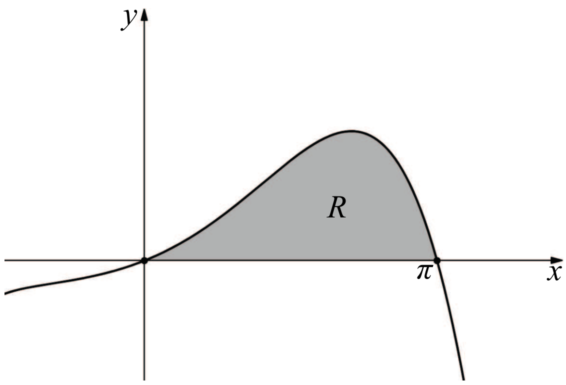
QUESTION THIRTY (3 marks) **Marks**



The curve shown in the diagram above has the equation $y = \dfrac{e^x \sin x}{x+2}$.

The finite region R bounded by the curve and the x-axis from $x = 0$ to $x = \pi$ is shown shaded in the diagram above.

(a) Complete the table below with the value of y corresponding to $x = \frac{\pi}{3}$, giving your answer correct to 5 decimal places. 1

x	0	$\dfrac{\pi}{3}$	$\dfrac{2\pi}{3}$	π
y	0		1.71761	0

(b) Use the trapezoidal rule, with all the values in the completed table, to obtain an estimate for $\displaystyle\int_0^{\pi} \frac{e^x \sin x}{x+2}\, dx$, the area of the region R. Give your answer correct to 4 decimal places. 2

QUESTION THIRTY-ONE (4 marks) **Marks**

The partial sums S_n of a geometric progression are given by $100, 190, 271, \ldots$

(a) By finding a formula for the nth term, T_n, find the value of n which gives the first term in the geometric progression smaller than 10. 3

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(b) Find the limiting sum, S_∞, of the geometric progression. 1

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QUESTION THIRTY-TWO (5 marks) **Marks**

A function is defined as $f(x) = \sin\left(2x + \frac{\pi}{2}\right) + 1$, $\quad 0 \leq x \leq 2\pi$.

(a) Sketch $y = f(x)$, clearly labelling any stationary points and intercepts with the axes. 3

(b) Hence, or otherwise, solve $\sin\left(2x + \frac{\pi}{2}\right) = 1$, $\quad 0 \leq x \leq 2\pi$. 2

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QUESTION THIRTY-THREE (9 marks) **Marks**

Let $f(x) = \dfrac{x^2 - 1}{x^3}$.

(a) State the natural domain of the function. 1

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(b) Show that the function is odd. 1

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It is given that $f'(x) = \dfrac{-x^2 + 3}{x^4}$.

(c) Show that $f''(x) = \dfrac{2x^2 - 12}{x^5}$. 1

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QUESTION THIRTY-THREE (Continued)

(d) Find any stationary points of $f(x)$ and determine their nature. [2]

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(e) Sketch the curve $y = \dfrac{x^2 - 1}{x^3}$, clearly labelling any stationary points, asymptotes, and intercepts with the axes. [4]

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CANDIDATE NUMBER

Section II

Part D

— Remember to fill in your candidate number above —

QUESTION THIRTY-FOUR (3 marks)

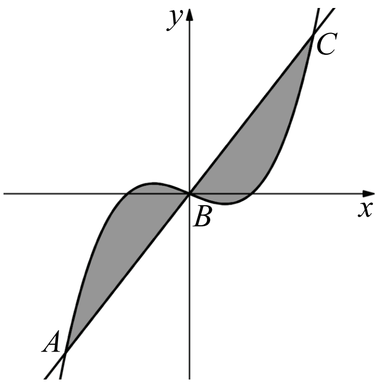
Marks

Solve $2\log_2(x+3) - \log_2 x = 4$. 3

QUESTION THIRTY-FIVE (4 marks)

Marks

4



Given $f(x) = x^3 - x$ and $g(x) = 3x$, find the area enclosed by $y = f(x)$ and $y = g(x)$, as shown by the shaded regions in the diagram above.

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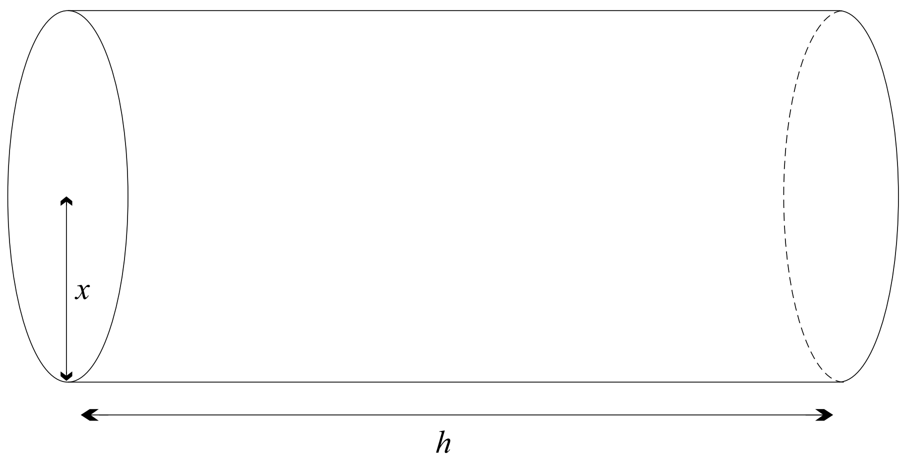
QUESTION THIRTY-SIX (6 marks) **Marks**



The diatium power cell is an integral component in all lightsabers. Each cell is cylindrical in shape, with a base radius x cm and height h cm, as shown in the diagram above.

(a) Given that the volume of each cell has to be $60\,\text{cm}^3$, show that the surface area, $A\,\text{cm}^2$, of the cell is given by $A = 2\pi x^2 + \dfrac{120}{x}$. 2

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QUESTION THIRTY-SIX (Continued)

(b) For safety reasons it is imperative that the surface area is as low as possible. Calculate the minimum surface area, correct to 2 decimal places, and justify your answer. 4

QUESTION THIRTY-SEVEN (5 marks) **Marks**

Bo-Katan and Din are racing with their jetpacks along a 2 km linear course. Bo-Katan accelerates at a constant $10\,\text{m/s}^2$ up to a top speed of $35\,\text{m/s}$, which she maintains until the finish. Din accelerates at a constant $8\,\text{m/s}^2$ up to a top speed of $40\,\text{m/s}$, which he maintains until the finish.

(a) Due to a technical malfunction, Din starts n seconds after Bo-Katan. Given that they both start from rest, and Din wins the race, what is the largest possible value of n? Give your answer correct to 2 decimal places. 3

QUESTION THIRTY-SEVEN (Continued)

(b) Suppose Din and Bo-Katan start at the same point at the same time, that is $n = 0$ seconds. [2]

At what displacement, x, from the start of the race will they meet again?

END OF PAPER

SYDNEY GRAMMAR SCHOOL

SOLUTIONS

NAME

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Checklist

- Reference sheet
- Multiple-choice answer sheet
- Candidature: 91 pupils

Writer: OMD

Section I

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Record the single best answer for each question on the provided answer sheet.

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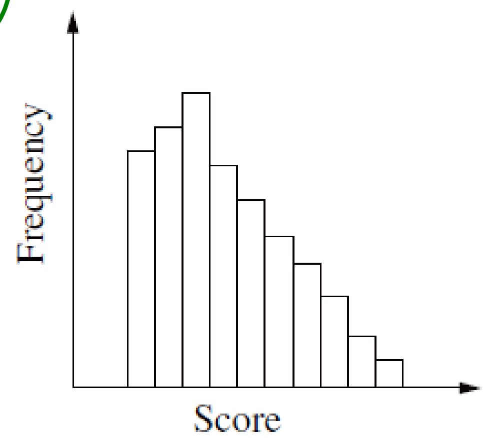
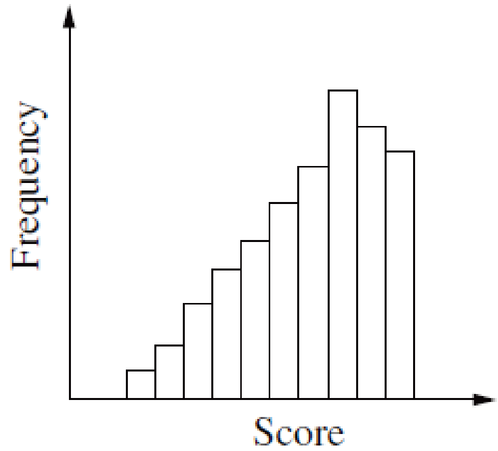
1. Which histogram best represents a dataset that is positively skewed?

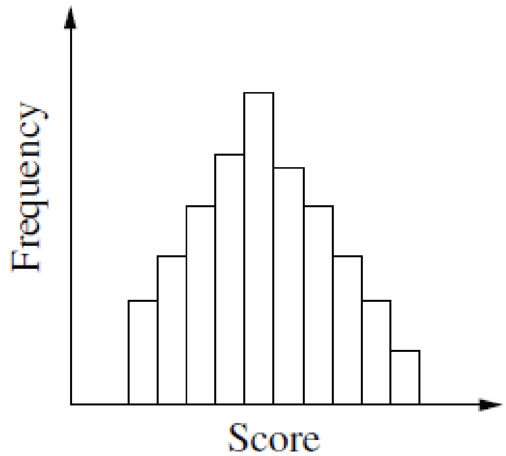
(A)



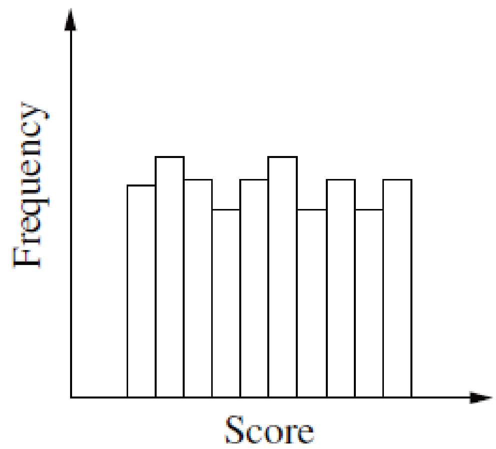
(B)



(C)



(D)



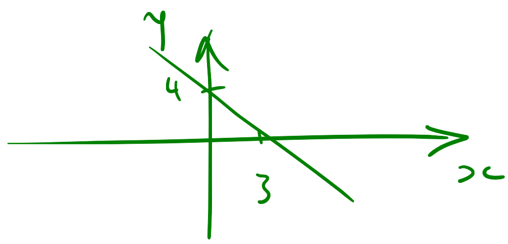
2. What is the gradient of the line passing through $A(3, 0)$ and $B(0, 4)$?

(A) $-\frac{4}{3}$

(B) $-\frac{3}{4}$

(C) $\frac{3}{4}$

(D) $\frac{4}{3}$



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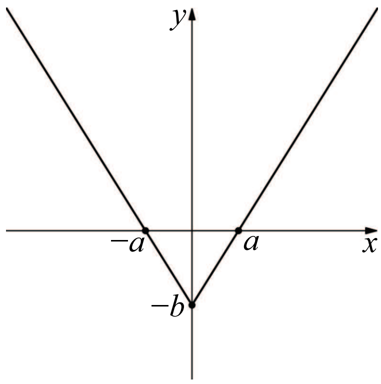


(A) Odd

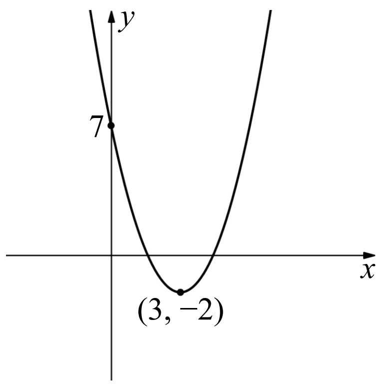
(B) Even

line symmetry $x = 0$.

(C) Neither

(D) Further information required

4. Which equation best describes the quadratic function sketched below?



(A) $y = (x + 3)^2 + 2$

(B) $y = (x + 3)^2 - 2$

(C) $y = (x - 3)^2 + 2$

(D) $y = (x - 3)^2 - 2$

right 3 down 2

5. In twelve years, the future value of an investment will be \$250 000. The interest rate is 6% per annum, compounded half-yearly.

   Which equation will give the present value (PV) of the investment?

   (A) $PV = \dfrac{250\,000}{(1+0.06)^{24}}$

   (B) $PV = \dfrac{250\,000}{(1+0.03)^{24}}$

   (C) $PV = \dfrac{250\,000}{(1+0.06)^{12}}$

   (D) $PV = \dfrac{250\,000}{(1+0.03)^{12}}$

half-yearly
12 yrs × 2 = 24 half years
6% per annum
$\frac{6\%}{2}$ = 3% per half year

6. Which transformations best describe the change from $y = f(x)$ to $y = -f(x+5)$?

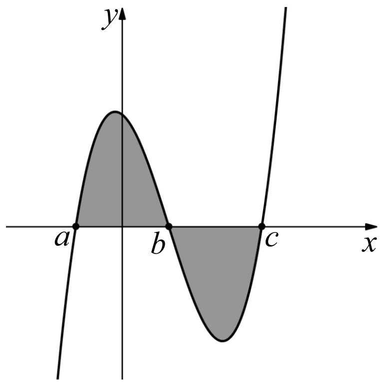
   (A) A translation left 5 units then a reflection in the y-axis.

   (B) A translation right 5 units then a reflection in the y-axis.

   (C) A translation left 5 units then a reflection in the x-axis.

   (D) A translation right 5 units then a reflection in the x-axis.

7. For the given function $y = f(x)$ sketched below which integral does **NOT** give the shaded area?



(A) $\int_a^b f(x)\,dx + \int_c^b f(x)\,dx$

(B) $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

(C) $\int_a^b f(x)\,dx - \int_b^c f(x)\,dx$

(D) $\int_a^b f(x)\,dx + \left|\int_b^c f(x)\,dx\right|$

8. For the random variable X, it is known that $\mathrm{E}(X) = 3$ and $\mathrm{Var}(X) = 2$. Which option below gives the correct expected value and variance of $X + 3$?

(A) $\mathrm{E}(X+3) = 3$ and $\mathrm{Var}(X+3) = 2$.

(B) $\mathrm{E}(X+3) = 6$ and $\mathrm{Var}(X+3) = 2$.

(C) $\mathrm{E}(X+3) = 3$ and $\mathrm{Var}(X+3) = 5$.

(D) $\mathrm{E}(X+3) = 6$ and $\mathrm{Var}(X+3) = 5$.

$E(aX+b) = aE(X) + b$

$Var(aX+b) = a^2 Var(X)$

9. Which is a correct trigonometric identity?

   (A) $(1 + \sec\theta)(1 - \sec\theta) = \cot^2\theta$

   (B) $(1 + \sec\theta)(1 - \sec\theta) = \tan^2\theta$

   (C) $(1 + \sec\theta)(1 - \sec\theta) = -\cot^2\theta$

   (D) $(1 + \sec\theta)(1 - \sec\theta) = -\tan^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 - \sec^2\theta = -\tan^2\theta$

$(1+\sec\theta)(1-\sec\theta) = -\tan^2\theta$

10. A region in the plane is bounded by the curve $y = \dfrac{1}{3x}$, the x-axis, the line $x = m$, and the line $x = 2m$, where $m > 0$. Which description most accurately describes the area of this region?

    (A) The area increases as m increases.

    (B) The area decreases as m increases.

    (C) The area increases as m decreases.

    (D) The area is independent of m.

$\frac{1}{3}\int_m^{2m} \frac{1}{x}\,dx$

$= \frac{1}{3}\left[\ln x\right]_m^{2m}$

$= \frac{1}{3}(\ln 2m - \ln m)$

$= \frac{1}{3}\ln\frac{2m}{m}$

$= \frac{1}{3}\ln 2$

End of Section I

The paper continues in the next section

QUESTION ELEVEN (2 marks) **Marks**

Find the equation of the line perpendicular to $y = 2x - 1$ passing through the point $A(0, -7)$. [2]

$m_1 = 2$ $\quad$ $y - -7 = -\frac{1}{2}(x - 0)$

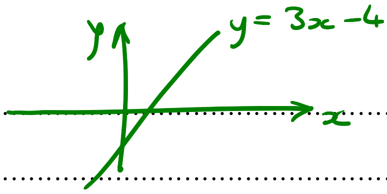
$m_2 = -\frac{1}{2}$ $\quad$ $y + 7 = -\frac{1}{2}x$

$y = -\frac{1}{2}x - 7$

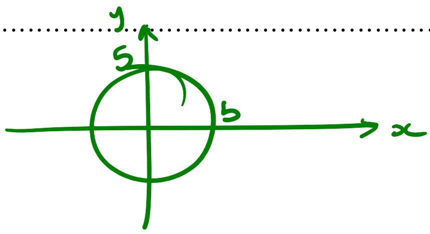
QUESTION TWELVE (2 marks) **Marks**

Classify each relation as one-to-one, many-to-one, one-to-many, or many-to-many.

(a) $y = 3x - 4$ [1]



One-to-one



(b) $x^2 + y^2 = 25$ [1]

Many-to-many

QUESTION THIRTEEN (1 mark) **Marks**

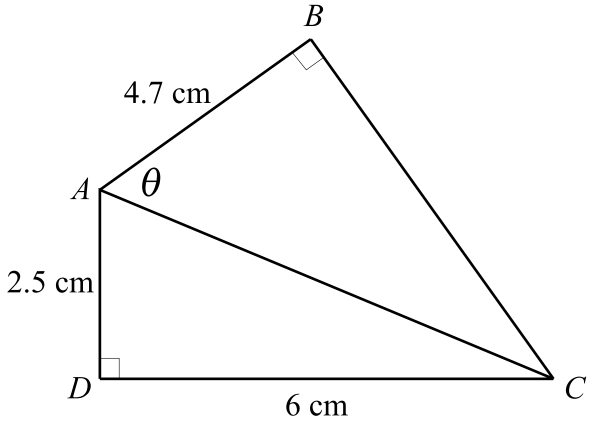
Differentiate $f(x) = x^2(2x - 1)$. [1]

$f(x) = 2x^3 - x^2$

$f'(x) = 6x^2 - 2x$

QUESTION FOURTEEN (3 marks) Marks

Two right-angled triangles, ABC and ADC, are shown below. 3



Calculate the size of angle θ. Give your answer correct to the nearest minute.

$AC = \sqrt{6^2 + 2.5^2}$

$= 6.5$ cm ✓

$\theta = \cos^{-1}\left(\frac{4.7}{6.5}\right)$ ✓

$= 43^\circ 41'$ ✓

QUESTION FIFTEEN (2 marks) Marks

Suppose that $f(x) = 2x^2$ and $g(x) = x - 1$.

(a) Find the value of $g(f(4))$. 1

$f(4) = 32$

$g(32) = 31$

$\therefore\ g(f(4)) = 31$ ✓

(b) Find $f(g(x))$. 1

$f(g(x)) = 2(x-1)^2$ accept factorised form

$= 2(x^2 - 2x + 1)$

$= 2x^2 - 4x + 2$ ✓

QUESTION SIXTEEN (2 marks) **Marks**

Find a formula for the nth term of the following sequences:

(a) $10, 12, 14, 16, \ldots$ 1

A.P. $T_n = a + (n-1)d \quad a = 10 \quad d = 2$

$= 10 + 2(n-1)$ ✓ accept

$= 2n + 8$

(b) $\frac{1}{3}, 1, 3, 9, \ldots$ 1

G.P. $T_n = ar^{n-1} \quad a = \frac{1}{3} \quad r = 3$

$= \frac{1}{3} \times 3^{n-1}$ ✓ accept

$= 3^{n-2}$

QUESTION SEVENTEEN (2 marks) **Marks**

Differentiate $y = (5x-3)^2$. 2

$y = (5x-3)^2$

$y' = 2 \times 5 \times (5x-3)$

$y' = 10(5x-3)$

QUESTION EIGHTEEN (2 marks) **Marks**

The gradient function of a curve is given by $f'(x) = 2x$. Given that the curve $y = f(x)$ goes through the point $A(2,-1)$, find the equation of the curve, $y = f(x)$. 2

$f'(x) = 2x$

$f(x) = x^2 + c$

$-1 = 2^2 + c$

$-1 = 4 + c$

$c = -5$

$f(x) = x^2 - 5$

QUESTION NINETEEN (4 marks) **Marks**

Daniel inherits \$50 000 and invests it in an account earning interest at a rate of 0.6% per month. Each month, immediately after the interest has been paid, Daniel withdraws \$700.

The amount in the account immediately after the nth withdrawal can be determined using the recurrence relation

$$A_n = A_{n-1}(1.006) - 700\,,$$

where $n = 1, 2, 3, \ldots$ and $A_0 = 50\,000$.

(a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal. **2**

$A_1 = 50000(1.006) - 700$ ✓
$= 49600$

$A_2 = 49600(1.006) - 700$
$= 49197.6$

$A_3 = 49197.6(1.006) - 700$
$= \$48792.79$ ✓

(b) Calculate the amount of interest earned in the first three months. **2**

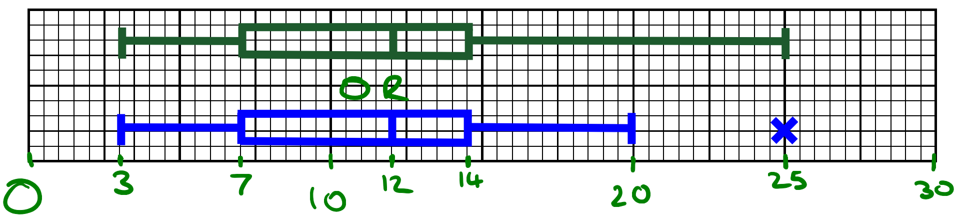
$I = A_3 + 3 \times 700 - 50000$ ✓ Allow ECF from Part (a)
$= \$892.79$ ✓ their $A_3 + 3 \times 700 - 50000$

QUESTION TWENTY (4 marks) **Marks**

The Jawas run a trading outpost on Tatooine. Their accountant records their monthly sales, in millions of galactic credits, listed below:

$$3, 4, 7, 9, 11, 12, 13, 13, 14, 20, 25\,.$$

(a) On the graph paper below, draw a box plot to represent this data. [2]



$Q_1 + Q_2 + Q_3$ ✓

whiskers ✓

3, 4, 7, 9, 11, 12, 13, 13, 14, 20, 25

Q_1 (7), Q_2 (12), Q_3 (14)

(b) Identify any outliers in this data. [1]

IQR = 14 − 7
= 7

Lower = 7 − 7 × 1·5
= −3·5
∴ no outliers at bottom end

Upper = 14 + 7 × 1·5
= 24·5
∴ 25 outlier ✓

(c) The Jawas claim that for 75% of the months, the amount received in sales is greater than 10 million galactic credits. Comment on this claim, justifying your answer. [1]

This claim is false. 75% of the months are above the lower quartile (Q_1) which is 7 million galactic credits. ✓

reference to Q_1 or 7 million galactic credits

QUESTION TWENTY-ONE (3 marks) **Marks**

(a) Find the equation of the tangent to the curve $y = e^{x+2}$ at the point $A(-2, 1)$. 2

$y' = e^{x+2}$

when $x = -2$

$y' = e^{-2+2}$

$= e^0$

$y' = 1$ ✓

$y - 1 = 1(x - -2)$

$y - 1 = x + 2$

$y = x + 3$ ✓

(b) Hence find point B, where the tangent in Part (a) crosses the x-axis. 1

when $y = 0$

$0 = x + 3$

$x = -3$

$(-3, 0)$

✓ accept either

QUESTION TWENTY-TWO (3 marks)

Marks

Solve the equation $9^x - 3^{x+1} = -2$, leave your answers in exact form. 3

$(3^x)^2 - 3(3^x) + 2 = 0$

Let $y = 3^x$

$y^2 - 3y + 2 = 0$ ✓ forms quadratic

$(y-2)(y-1) = 0$

$\therefore$ EITHER $y - 2 = 0$ OR $y - 1 = 0$ ✓

$y = 2$ $y = 1$

$3^x = 2$ $3^x = 1$

$x = \log_3 2$ $x = 0$

$\doteqdot 0.6309$ ✓

QUESTION TWENTY-THREE (2 marks) **Marks**

Find the sum of the first 100 positive odd numbers. 2

A.P. $a = 1$ $d = 2$ $n = 100$

$S_{100} = \frac{100}{2}\left[2 \times 1 + (100 - 1) \times 2\right]$ ✓

$= 10\,000$ ✓

QUESTION TWENTY-FOUR (2 marks) **Marks**

Differentiate $y = \dfrac{x^2 + 1}{e^{2x}}$, leave your answer in simplest form. 2

$y = \frac{u}{v}$ $u = x^2 + 1$ $v = e^{2x}$

$u' = 2x$ $v' = 2e^{2x}$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{2xe^{2x} - 2e^{2x}(x^2+1)}{e^{4x}}$ ✓

$= \frac{2x - 2(x^2+1)}{e^{2x}}$

$= \frac{2x - 2x^2 - 2}{e^{2x}}$ ✓

QUESTION TWENTY-FIVE (4 marks) **Marks**

The table shows the future value of an annuity of \$1.

Future values of an annuity of \$1

Years	Interest Rate per Annum			
	1%	2%	3%	4%
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

Harper is saving for a trip and estimates she will need \$20 000. She opens an account earning 4% per annum, compounded annually.

(a) How much does Harper need to deposit every year if she wishes to have enough money for the trip in 5 years time? 2

FV. = 20000 ÷ (5·416) ✓

= \$3692·76 ✓ allow ECF

(b) How much interest will Harper earn on her investment over the 5 years? Give your answer correct to the nearest dollar. 2

I = 20000 − 5 × 3692·76 ✓

= \$1536 ✓ allow ECF

QUESTION TWENTY-SIX (3 marks) **Marks**

(a) Find $\int e^{3-2x}\,dx$. 1

$-\frac{1}{2}\int -2e^{3-2x}\,dx$

$-\frac{1}{2}e^{3-2x}$ $(+C)$ ✓

(b) Evaluate $\int_0^1 \frac{1}{3x+1}\,dx$, leave your answer in exact form. 2

$\frac{1}{3}\int_0^1 \frac{3}{3x+1}\,dx$

$= \frac{1}{3}\left[\ln|3x+1|\right]_0^1$ ✓

$= \frac{1}{3}(\ln 4 - \ln 1)$

$= \frac{1}{3}\ln 4$ ✓

QUESTION TWENTY-SEVEN (3 marks) **Marks**

Gizka have proven to be a pest throughout the galaxy due to their exponential rate of reproduction. The number, G, of gizka at a time, t, months after they were introduced to the planet of Tatooine can be calculated by the formula $G = 50e^{1.2t}$.

(a) How many gizka were first released on Tatooine? 1

when $t = 0$

$G = 50$ ✓

(b) Show that $\frac{dG}{dt} = 1.2G$ and hence find the rate at which the number of gizka was increasing when there were 200 gizka. 2

$LHS = \frac{dG}{dt}$

$= 50e^{1.2t} \times 1.2$

$= 60e^{1.2t}$

$= RHS$

$RHS = 1.2G$

$= 1.2 \times 50e^{1.2t}$

$= 60e^{1.2t}$

✓ show that

$\frac{dG}{dt} = 1.2 \times 200$

$= 240$ gizka per month ✓

QUESTION TWENTY-EIGHT (2 marks) **Marks**

(a) Differentiate $\cos^3 x$ with respect to x. 1

$y = \cos^3 x$ $\qquad u = \cos x$

$= u^3$ $\qquad \frac{du}{dx} = -\sin x$

$\frac{dy}{du} = 3u^2$

$= 3\cos^2 x$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= -3\sin x \cos^2 x$ ✓

(b) Hence, or otherwise, find $\displaystyle\int \sin x \cos^2 x \, dx$. 1

$-\frac{1}{3}\int -3\sin x \cos^2 x \, dx$

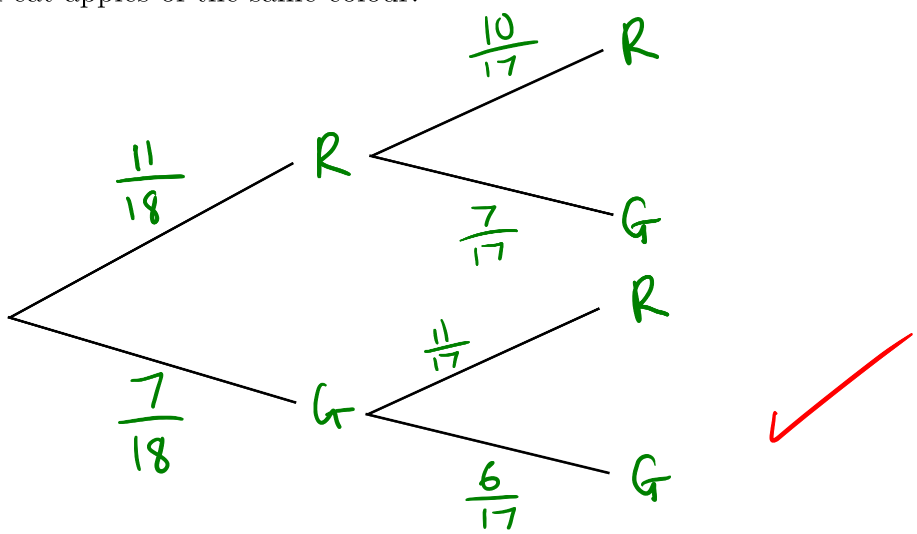
$= -\frac{1}{3}\cos^3 x + C$ ✓

QUESTION TWENTY-NINE (5 marks) **Marks**

A bowl of fruit contains 18 apples of which 11 are red and 7 are green.

Matthew takes one apple at random and eats it. Evelyn then takes an apple at random and eats it.

(a) By drawing a probability tree diagram, or otherwise, find the probability that Matthew and Evelyn eat apples of the same colour. 3



$P(\text{SAME}) = P(RR) + P(GG)$

$= \frac{11}{18} \times \frac{10}{17} + \frac{7}{18} \times \frac{6}{17}$ ✓ P(RR) or P(GG)

$= \frac{76}{153}$ ✓ allow ft from incorrect tree diagram.

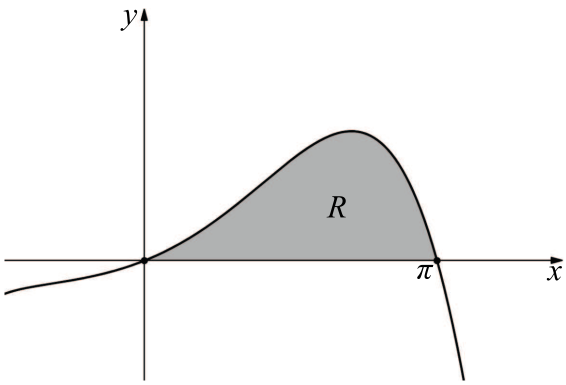
(b) Given that Matthew and Evelyn eat apples of the same colour, find the probability that they are red. 2

$P(RR \mid \text{SAME}) = \frac{P(RR)}{P(\text{SAME})}$ ✓

$= \frac{55}{153} \div \frac{76}{153}$

$= \frac{55}{76}$ ✓

QUESTION THIRTY (3 marks) **Marks**



The curve shown in the diagram above has the equation $y = \dfrac{e^x \sin x}{x+2}$.

The finite region R bounded by the curve and the x-axis from $x = 0$ to $x = \pi$ is shown shaded in the diagram above.

(a) Complete the table below with the value of y corresponding to $x = \frac{\pi}{3}$, giving your answer correct to 5 decimal places. 1

DEG MODE = 0.01709122979

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π
y	0	0.80988	1.71761	0

(b) Use the trapezoidal rule, with all the values in the completed table, to obtain an estimate for $\displaystyle\int_0^\pi \frac{e^x \sin x}{x+2}\,dx$, the area of the region R. Give your answer correct to 4 decimal places. 2

$R = \dfrac{\pi - 0}{2 \times 3}\left[0 + 0 + 2\left(0.80988 + 1.71761\right)\right]$ ✓ correct use of formula

$= 2.646784199$ with unrounded value of $f\left(\frac{\pi}{2}\right)$

$= 2.646781339$ with rounded value of $f\left(\frac{\pi}{2}\right)$

$\doteqdot 2.6468$ (4 d.p.) ✓ rounding

DEG MODE value gives $\doteqdot 1.81657488$

✓✓ ECF

$\doteqdot 1.8166$ (4 d.p.)

QUESTION THIRTY-ONE (4 marks) **Marks**

The partial sums S_n of a geometric progression are given by $100, 190, 271, \ldots$

(a) By finding a formula for the nth term, T_n, find the value of n which gives the first term in the geometric progression smaller than 10. 3

$T_1 = 100$

$T_2 = 190 - 100 = 90$

$T_3 = 271 - 190 = 81$

G.P. $a = 100$ $r = 0.9$

$T_n = 100 \times 0.9^{n-1}$

$10 > 100 \times 0.9^{n-1}$

$0.1 > 0.9^{n-1}$

$\log(0.1) > (n-1)\log(0.9)$

$\frac{\log(0.1)}{\log(0.9)} < n - 1$

$n > \frac{\log 0.1}{\log 0.9} + 1$

$n > 22.85$

$\therefore n = 23$

(b) Find the limiting sum, S_∞, of the geometric progression. 1

$S_\infty = \frac{100}{1 - 0.9}$

$= 1000$

QUESTION THIRTY-TWO (5 marks) **Marks**

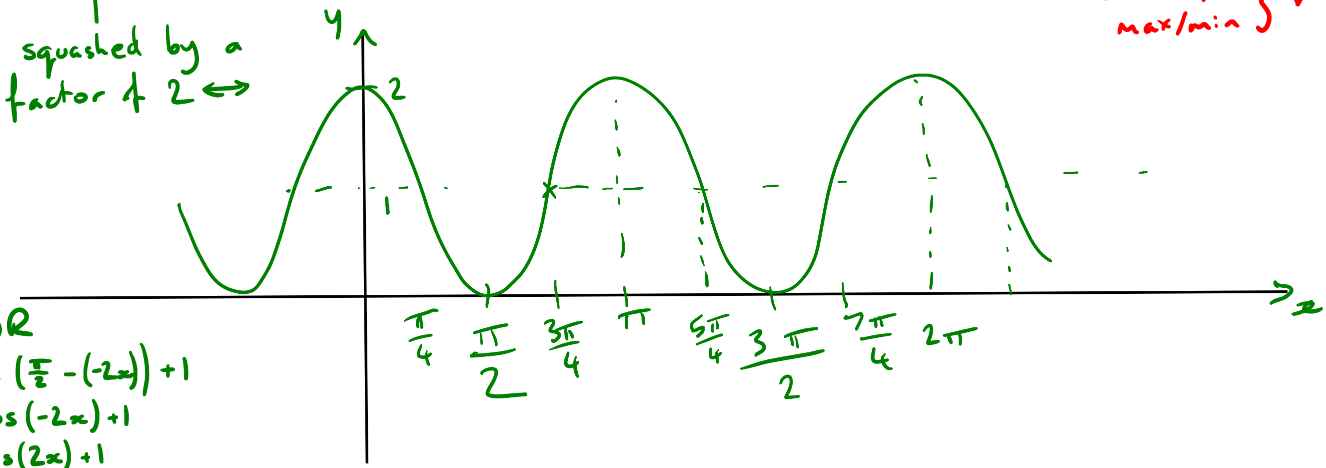
A function is defined as $f(x) = \sin\left(2x + \frac{\pi}{2}\right) + 1\,, \qquad 0 \le x \le 2\pi\,.$

(a) Sketch $y = f(x)$, clearly labelling any stationary points and intercepts with the axes. 3

$f(x) = \sin\left(2\left(x + \frac{\pi}{4}\right)\right) + 1$

left $\frac{\pi}{4}$; up 1; squashed by a factor of 2 ⟷

$0 \le y \le 2$ ✓
period $= \pi$ ✓
intercepts, max/min ✓



OR

$f(x) = \sin\left(\frac{\pi}{2} - (-2x)\right) + 1$
$= \cos(-2x) + 1$
$= \cos(2x) + 1$
as cosine is EVEN function

(b) Hence, or otherwise, solve $\sin\left(2x + \frac{\pi}{2}\right) = 1\,, \qquad 0 \le x \le 2\pi\,.$ 2

$0 \le 2x \le 4\pi$
$\frac{\pi}{2} \le 2x + \frac{\pi}{2} \le \frac{9\pi}{2}$

$\sin\left(2x + \frac{\pi}{2}\right) = 1 \quad (+1)$

$\sin\left(2x + \frac{\pi}{2}\right) + 1 = 2$

$\therefore$ where graph $f(x) = 2$ ✓

$x = 0, \pi, 2\pi$ ✓

allow ECF

$2x + \frac{\pi}{2} = \sin^{-1} 1$
$= \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$ ✓

$2x = 0, 2\pi, 4\pi$

$x = 0, \pi, 2\pi$ ✓

graphical solution OR algebraic solution

QUESTION THIRTY-THREE (9 marks) **Marks**

Let $f(x) = \dfrac{x^2 - 1}{x^3}$.

(a) State the natural domain of the function. 1

$x \neq 0$ ✓

(b) Show that the function is odd. 1

$-f(x) = -\dfrac{x^2-1}{x^3}$

$f(-x) = \dfrac{(-x)^2 - 1}{(-x)^3}$

$= \dfrac{x^2 - 1}{-x^3}$

$= -\dfrac{x^2-1}{x^3}$

$-f(x) = f(-x)$ ∴ ODD ✓ show that

It is given that $f'(x) = \dfrac{-x^2 + 3}{x^4}$.

(c) Show that $f''(x) = \dfrac{2x^2 - 12}{x^5}$. 1

$f'(x) = -x^{-2} + 3x^{-4}$

$f''(x) = 2x^{-3} - 12x^{-5}$

$= \dfrac{2x^2 - 12}{x^5}$ As required ✓ show that

QUESTION THIRTY-THREE (Continued)

(d) Find any stationary points of $f(x)$ and determine their nature. [2]

$f'(x) = \dfrac{(\sqrt{3}+x)(\sqrt{3}-x)}{x^4}$ s.p. when $x = \sqrt{3}$ or $-\sqrt{3}$

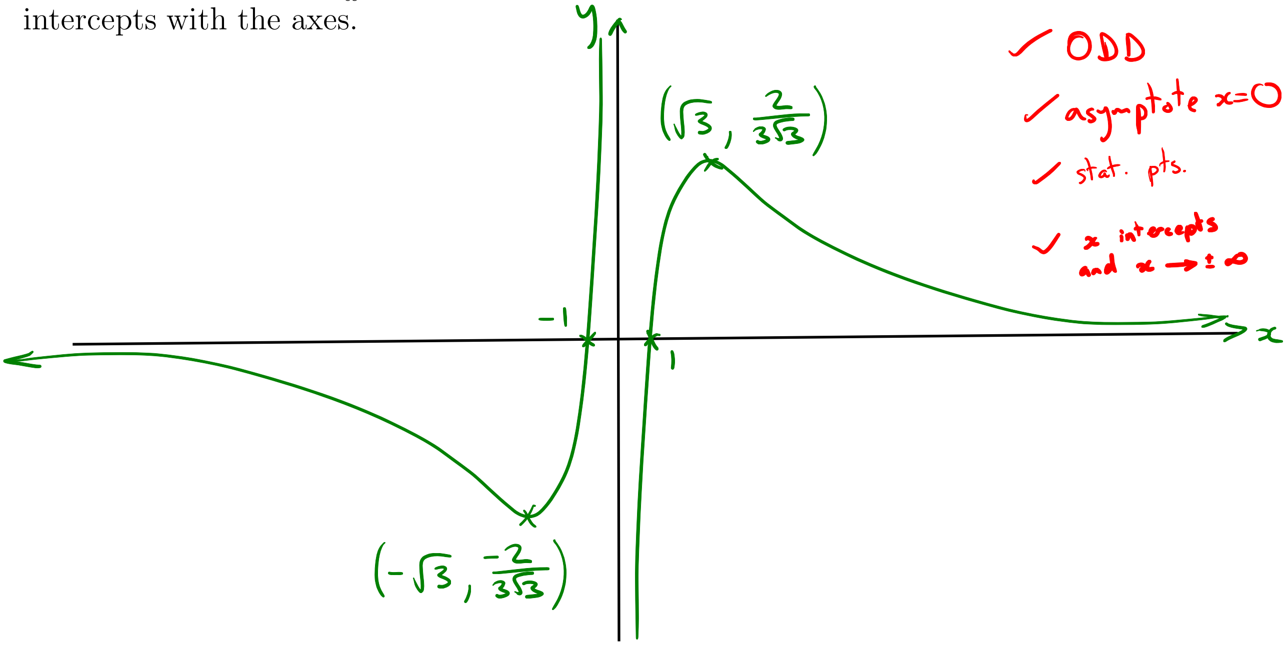
$y = \dfrac{2}{3\sqrt{3}}$ $y = \dfrac{-2}{3\sqrt{3}}$

$f''(\sqrt{3}) = \dfrac{-6}{9\sqrt{3}}$ $f''(-\sqrt{3}) = \dfrac{6}{9\sqrt{3}}$

< 0 > 0

$\therefore \left(\sqrt{3}, \dfrac{2}{3\sqrt{3}}\right)$ MAX $\left(-\sqrt{3}, \dfrac{-2}{3\sqrt{3}}\right)$ MIN

(e) Sketch the curve $y = \dfrac{x^2-1}{x^3}$, clearly labelling any stationary points, asymptotes, and intercepts with the axes. [4]



as $x \to \infty$
$y \to 0^+$

as $x \to -\infty$
$y \to 0^-$

as $x \to 0^-$
$y \to \infty$

as $x \to 0^+$
$y \to -\infty$

QUESTION THIRTY-FOUR (3 marks) **Marks**

Solve $2\log_2(x+3) - \log_2 x = 4$. 3

$$\log_2\left(\frac{(x+3)^2}{x}\right) = 4$$ ✓

$$\frac{(x+3)^2}{x} = 2^4$$

$$(x+3)^2 = 16x$$

$$x^2 + 6x + 9 = 16x$$

$$x^2 - 10x + 9 = 0$$ ✓

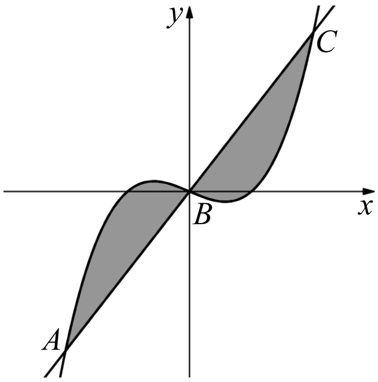
$$(x-9)(x-1) = 0$$

$$x = 9 \quad \text{or} \quad x = 1$$ ✓

QUESTION THIRTY-FIVE (4 marks)

Marks

4



Given $f(x) = x^3 - x$ and $g(x) = 3x$, find the area enclosed by $y = f(x)$ and $y = g(x)$, as shown by the shaded regions in the diagram above.

intersect when

$f(x) = g(x)$

$x^3 - x = 3x$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x(x+2)(x-2) = 0$

$\therefore$ EITHER $x = 0$ $x = -2$ $x = 2$ ✓

both functions odd $\therefore$ $A = 2\int_0^2 3x - (x^3 - x)\,dx$ ✓

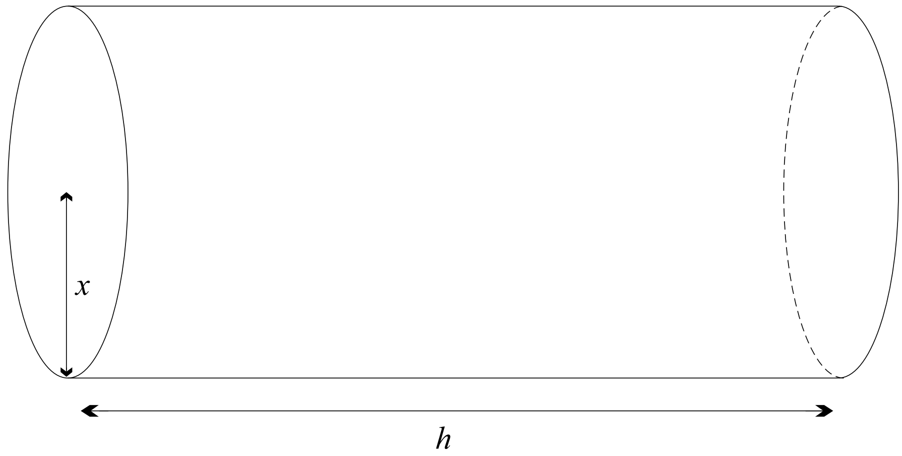
$= 2\int_0^2 4x - x^3\,dx$

$= 2\left[2x^2 - \frac{x^4}{4}\right]_0^2$ ✓

$= 2(8 - 4) - 0$

$= \underline{8 \text{ units}^2}$ ✓

QUESTION THIRTY-SIX (6 marks) **Marks**



The diatium power cell is an integral component in all lightsabers. Each cell is cylindrical in shape, with a base radius x cm and height h cm, as shown in the diagram above.

(a) Given that the volume of each cell has to be $60\,\text{cm}^3$, show that the surface area, $A\,\text{cm}^2$, of the cell is given by $A = 2\pi x^2 + \dfrac{120}{x}$. [2]

$V = \pi r^2 h$

$60 = \pi x^2 h$

$h = \dfrac{60}{\pi x^2}$ ✓

$A = 2\pi r^2 + 2\pi r h$

$= 2\pi x^2 + 2\pi x \times \dfrac{60}{\pi x^2}$ ✓ show that

$A = 2\pi x^2 + \dfrac{120}{x}$ As required.

QUESTION THIRTY-SIX (Continued)

(b) For safety reasons it is imperative that the surface area is as low as possible. Calculate the minimum surface area, correct to 2 decimal places, and justify your answer. 4

$A = 2\pi x^2 + 120x^{-1}$

$\frac{dA}{dx} = 4\pi x - 120x^{-2}$ ✓ $\qquad \frac{d^2A}{dx^2} = 4\pi + 240x^{-3}$

s.p. where $\frac{dA}{dx} = 0$

$4\pi x = \frac{120}{x^2} \qquad x \neq 0$

$x^3 = \frac{120}{4\pi}$

$x = \sqrt[3]{\frac{30}{\pi}}$ ✓

$\frac{d^2A}{dx^2} = 4\pi + \frac{240}{\frac{120}{4\pi}}$

$\doteqdot 37.699$

> 0

$\therefore$ MIN ✓

$A = 2\pi\left(\sqrt[3]{\frac{30}{\pi}}\right)^2 + \frac{120}{\sqrt[3]{\frac{30}{\pi}}}$

$\doteqdot 84.84 \text{ cm}^2$ ✓

QUESTION THIRTY-SEVEN (5 marks) **Marks**

Bo-Katan and Din are racing with their jetpacks along a 2 km linear course. Bo-Katan accelerates at a constant 10 m/s^2 up to a top speed of 35 m/s, which she maintains until the finish. Din accelerates at a constant 8 m/s^2 up to a top speed of 40 m/s, which he maintains until the finish.

(a) Due to a technical malfunction, Din starts n seconds after Bo-Katan. Given that they both start from rest, and Din wins the race, what is the largest possible value of n? Give your answer correct to 2 decimal places. **3**

Bo-Katan $\frac{35}{10} = 3.5$

$a = 10$ $\quad 0 \le t \le 3.5$

$v = 10t + c_1$ $\quad 0 \le t \le 3.5$

start from rest $c_1 = 0$

$v = 10t$ $\quad 0 \le t \le 3.5$

$x = 5t^2 + c_2$

start from origin $c_2 = 0$

$x = 5t^2$ $\quad 0 \le t \le 3.5$

$v = 35$ $\quad 3.5 < t$

$x = 35(t-3.5) + c_3$ $\quad 3.5 < t$

when $t = 3.5$, $x = 5(3.5)^2 = 61.25$

$x_{BK} = 35(t-3.5) + 61.25$ $\quad 3.5 < t$

$= 35t - 122.5 + 61.25$

$= 35t - 61.25$ ✓

Din $\frac{40}{8} = 5$

$a = 8$ $\quad 0 \le t \le 5$

$v = 8t + c_4$ $\quad 0 \le t \le 5$

start from rest $c_4 = 0$

$v = 8t$ $\quad 0 \le t \le 5$

$x = 4t^2 + c_5$

start from origin $c_5 = 0$

$x = 4t^2$ $\quad 0 \le t \le 5$

$v = 40$ $\quad 5 < t$

$x = 40(t-5) + c_6$ $\quad 5 < t$

when $t = 5$, $x = 4(5)^2 = 100$

$x_D = 40(t-5) + 100$ $\quad 5 < t$

$= 40t - 200 + 100$

$= 40t - 100$ ✓

$2000 = 35t - 61.25$

$t = \frac{2000 + 61.25}{35}$

$\doteqdot 58.89\ldots$

$2000 = 40t - 100$

$t = \frac{2000+100}{40}$

$= 52.5$

$n = 58.89 - 52.5$

$= 6.39$ s ✓

Alternate geometric approach below...

QUESTION THIRTY-SEVEN (Continued)

(b) Suppose Din and Bo-Katan start at the same point at the same time, that is $n = 0$ seconds. [2]

At what displacement, x, from the start of the race will they meet again?

In same place when $x_{BK} = x_D$

$35t - 61.25 = 40t - 100$

$38.75 = 5t$

$t = 7.75\text{ s}$ ✓

$x = 40(7.75 - 5) + 100$

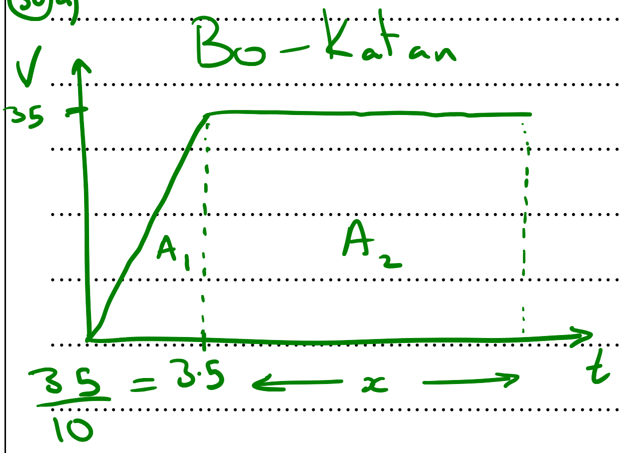
$= 210\text{ m}$ ✓

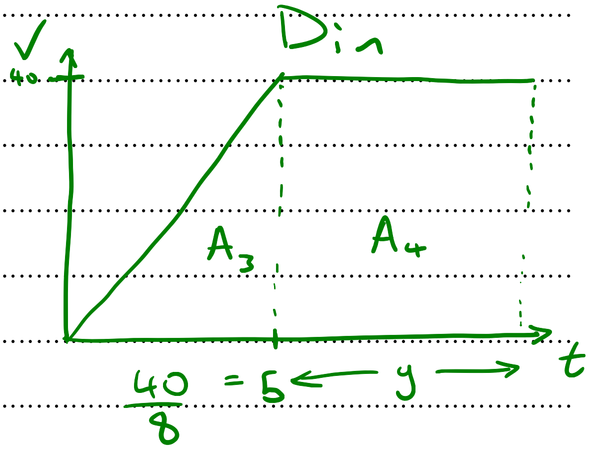
END OF PAPER

Extra writing space (Use this space only for questions in Part D)

If you use this space, clearly indicate which question you are answering.

36 a)





$A_1 = \frac{3.5 \times 35}{2}$

$= 61.25$

$A_2 = 2000 - 61.25$

$= 1938.75$

$x = \frac{1938.75}{35}$

$\doteqdot 55.39$

$\therefore$ Bo-Katan takes $55.39 + 3.5$

$= 58.89$ s ✓

$A_3 = \frac{5 \times 40}{2}$

$= 100$

$A_4 = 2000 - 100$

$= 1900$

$y = \frac{1900}{40}$

$y = 47.5$

$\therefore$ Din takes $47.5 + 5$

$= 52.5$ s ✓

$\therefore t = 58.89 - 52.5$

$= 6.39$ s ✓